

Project Title: On the Hopf-Thom problem for orientable surfaces.

Abstract:

The object of my research was to study extending a given immersion of a curve to an immersion of a smooth compact surface with a closed connected curve boundary. My approach to this study lied in hammering out the precise definitions necessary and, though I did not complete my study in the given time-frame, I learned a great amount about notions in various mathematical fields, particularly Topology. The most relevant to my study were several notions of topological invariants, properties, such as the Euler characteristic and the rotation number, which are preserved under homeomorphisms. These invariants allowed me to compare two spaces and study how these spaces changed after performing operations on them. Another notion of great importance to this study was how surfaces were classified, by being either orientable or non-orientable, consisting of the connected sum of torii or the connected sum of the real projective plane or a combination of both. Properties of the non-orientable surfaces allowed me to disregard immersions of these surfaces in the latter half of my study, which I had not assumed to be the case initially.

I am highly aware of the applications of the topics I have learned about, particularly how they underlie the various branches of physics. In addition, this opportunity was extremely benefitting to me. Not only did I learn a great deal about the subjects involved but I have also gotten a taste of the frustratingly precise rigor demanded by mathematicians, which will no doubt be helpful to me in the near future as I pursue further opportunities.