I-Center Research Abstract

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June 21, 2018

In this project, Gabe Kerr and I continue in our exploration of the equivalence of categories between 2-dimensional topological quantum field theories (TQFTs) and commutative Frobenius algebras as outlined in the short versions of Joachim Kock's book *Frobenius algebras and 2D topological quantum field theories* from the previous semester. Physicists take interest in this because TQFTs have traits resembling many aspects of quantum gravity theory. Mathematicians take interest because they are a useful tool in finding invariants of closed manifolds. In reverse, though, we can discover properties of TQFTs from properties of Frobenius algebras.

The bulk of our project takes us far into the second semester of research as well, where we try to answer the question of what Frobenius forms look like on Milnor rings with underlying fields (denoted $\mathbb{K}$) of arbitrary characteristic. In particular, we want to know whether we can obtain any new invariants of isolated singularities using such Frobenius forms. Therefore, we consider a sort of "residue map" on the Milnor ring in n variables, $M(f)$, given by $g \mapsto (x_1 f_{x_1}^{-1} x_2 f_{x2}^{-1} ... x_n f_{x_n}^{-1})g$ and then taking the constant term, where $g \in M(f) = {}^{\mathbb{K}[[x]]}\!\big/_{J(f)}$ and each $f_{x_i}^{-1}$ is the inverse of each generator of the Jacobian ideal, $J(f)$. The primary result from the project the first semester is that this is indeed a well-defined map on the Milnor ring, which we prove by induction on the number of variables of the Milnor ring. To show this is a Frobenius map is turned out to be too difficult for the scope and time frame of this project, despite or ambitions. Consequently, we have developed a solid survey of the categorical equivalence of TQFTs and Frobenius algebras and a decent example of its applications in singularity theory.