Project: A fast numerical solver for a scattering problem for inhomogeneous media

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Abstract

The focus of this project lies in the study of a computational algorithm for solving a scattering problem in two dimensions. This scattering problem belongs to the field of scattering theory, which has been used in many applications including geophysical prospecting, medical imaging, optics and nondestructive testing. When an object is illuminated by some incident wave, it produces scattered waves. The scattering problem aims to find the scattered wave given knowledge of the incident wave and the scattering object. This problem can be mathematically described by the Lippmann-Schwinger equation. Gennadi Vainikko developed in his paper, published in 2000, an efficient computational algorithm for solving the Lippmann-Schwinger equation. However, the algorithm presented in his paper is only valid for wave number equals 1.

Our goal in this project is to develop a MATLAB code for a modified version of Vainikko's algorithm that can work for any (positive) wave number. Obviously since our code can work for any wave number, it is more applicable than the original version from Vainikko's paper. We have tested the code for the following case: an inhomogeneous ball is illuminated by an incident plane wave propagating along the horizontal axis. The small errors in the solution for different levels of approximation in Figure 1 shows that the code works fine for different wave numbers. Figure 2 presents an example of the incident field and the scattered field calculated by our code.

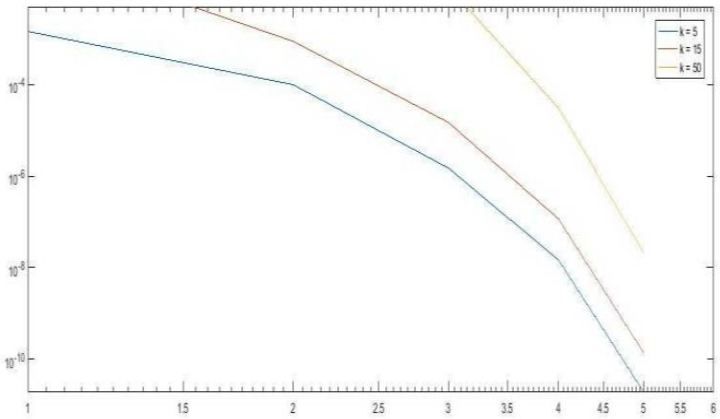

Figure 1: The percent error lines plotted on a graph for different wave numbers k = 5, 15,50.

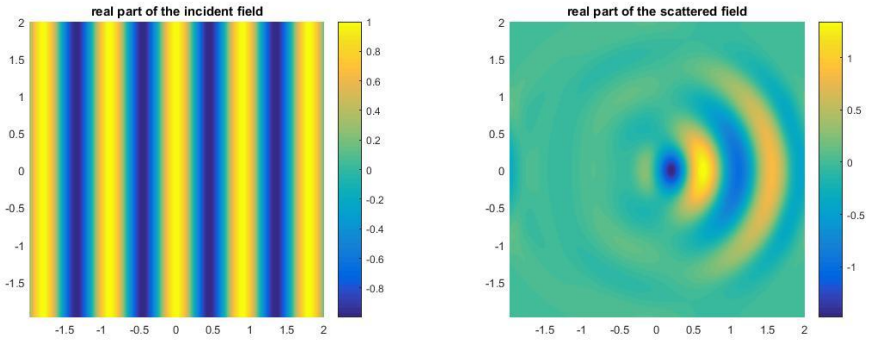

Figure 2: These plotted images were obtained from our MATLAB code. They represent the real part of the incident wave and the real part of the scattered wave.